

# A new temporal interpolation method for high-frequency vector wind fields

Jorge Zavala-Hidalgo, Mark A Bourassa, Steven L. Morey, James J. O'Brien, and Peng Yu  
 COAPS/Florida State University  
 Tallahassee, FL 32306-2840, USA  
 zavala@coaps.fsu.edu

**Abstract** – A new technique for the time interpolation of the forcing fields that recovers the movement of propagating features is introduced and applied to vector wind fields. The method involves the decomposition of the fields into their complex empirical orthogonal functions, and the interpolation of the temporal functions for the significant modes. The technique is tested using atmospheric model vector wind fields sampled at coarse temporal resolution to demonstrate the recovery of the wind fields at the intermediate times. The technique is also applied to a gridded vector wind product from satellite scatterometer data.

## I. INTRODUCTION

Several geophysical products are available for specific frequencies; examples are the synoptic data, the NCEP-Reanalysis, and available model output. For some applications may be desirable to have those data at higher frequencies, for example the new generation of very high-resolution ocean numerical models that allow the study of processes close to the inertial frequency and even in superinertial frequencies. These processes require that the models be forced with high frequency forcing fields that, in many cases, are obtained from atmospheric numerical models, typically in 6-hour fields. These fields commonly represent the values of surface fluxes of wind stress, evaporation, precipitation, and heat at specific dates and times. Oceanographers usually linearly interpolate these fields in time to the model time step. This is a good approximation for features that behave like standing waves, i.e., change their amplitude without moving. When there are fast moving features, such as fronts, the interpolated fields behave like duplicated standing fronts, changing their amplitude instead of moving. In this study, a new technique for time interpolation of fields characterized by moving features is presented. The technique is tested using synthetic data and a realistic 12-hour wind field to reconstruct a 1-hour field. This method decomposes the fields into their propagating empirical orthogonal functions, then the information from each mode is used to interpolate in time, and finally the significant modes are added.

## II. METHODOLOGY

The main goal is to have a good interpolation in time recovering moving features such as fronts. Propagating or complex empirical orthogonal functions (CEOFs) extract information from a two-dimensional data array  $H = (h_{nm})$ , where  $H$  is an  $N \times M$  matrix of elements  $h_{nm}$ , and  $N$  and  $M$  are

the number of space and time points, respectively. Applying CEOFs to  $D$ , it is decomposed in eigenmodes. For a detailed discussion of CEOFs see [1, 2].

The eigenmodes of  $H$  consist of complex vectors

$$T_i = (T_m) \text{ and } S_i = (S_n)$$

where vector  $S$  is the spatial function and  $T$  is the temporal function. By definition, any complex number  $C(x)$  may be written as  $C(x) = A(x)\exp[i\theta(x)]$ , where  $A(x)$  is the amplitude or magnitude of the vector and  $\theta(x)$  is the phase. Using this identity, the spatial and temporal functions may be represented as:

$$\begin{aligned} T(t) &= R(t)\exp(i\phi(t)) \text{ ,} \\ S(x) &= E(x)\exp(i\theta(x)) \text{ .} \end{aligned}$$

The real part of their product (ST)

$$\text{Re}\{E(x)R(t)\exp[i\theta(x)+\phi(t)]\} = E(x)R(t)\cos[\theta(x)+\phi(t)]$$

is the reconstructed field, where  $E(x)$  is the variability of the amplitude in space associated with a given eigenmode, and  $R(t)$  is the temporal amplitude function. The term  $[\theta(x)+\phi(t)]$  represents the phase at a given position and time. In our case, we are interested in the values at different times than the sampling ones but at the same location, therefore we take  $R$  and  $\phi$  at the sampling times and interpolate them into the desired times; then we add each mode.

This technique can be applied to any variable or combination of variables. For the interpolation of wind stress the technique is tested using two experiments: one decomposing the wind stress fields into scalar fields and then going back to the wind stress fields, and the other one directly applying the CEOFs decomposition to the wind stress fields.

### A. Wind stress reconstruction from scalar fields

In the first experiment the vorticity, divergence, and deformation terms, as well as the CEOF for each one, are computed, a 1-hour field is build, and the wind stress field is recovered from the scalar fields.

The decomposition of a linear wind field can be expressed as follows [3]:

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} u_0 \\ v_0 \end{pmatrix} + \begin{pmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{pmatrix}_0 \begin{pmatrix} x \\ y \end{pmatrix}$$

where  $u$  and  $v$  are the  $x$  and  $y$  components of the horizontal wind field  $\mathbf{V}$ , and  $u_0$  and  $v_0$  are the values of  $u$  and  $v$  at the origin. The above expression may be written as

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} u_0 \\ v_0 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} \delta & 0 \\ 0 & \delta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 0 & -\zeta \\ \zeta & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \frac{1}{2} \begin{pmatrix} D_1 & D_2 \\ D_2 & -D_1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \quad (1)$$

where

$$\delta = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \nabla_h \cdot \mathbf{V}$$

is the horizontal divergence,

$$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \hat{\mathbf{k}} \cdot \nabla \times \mathbf{V},$$

is the relative vertical vorticity, and

$$D_1 = \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}, \quad D_2 = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y},$$

are the deformation terms.

First derivatives of the horizontal wind components with respect to space are used to compute divergence, vorticity, and deformation terms. Even if the wind field is nonlinear, it is nearly linear in the neighborhood of any point at which computations of  $\delta$ ,  $\zeta$ , and  $D$  are made.

The first term in the right hand side of equation (1) represents uniform translation near the origin,  $u = u_0$  and  $v = v_0$ . In a field of pure translation there is no divergence, vorticity, or deformation.

The second term represents expansion ( $\delta > 0$ ) or contraction ( $\delta < 0$ ) about the origin. A field of pure divergence, or irrotational, is given by this term alone as

$$u = \frac{1}{2}\delta x, \quad v = \frac{1}{2}\delta y.$$

The third term represents counterclockwise ( $\zeta > 0$ ) or clockwise ( $\zeta < 0$ ) rotation about the origin. A field of pure vorticity is given by this term alone as

$$u = -\frac{1}{2}\zeta y, \quad v = \frac{1}{2}\zeta x.$$

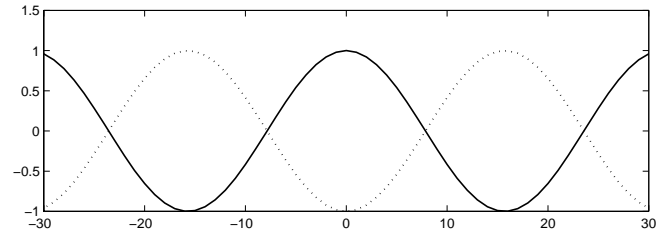
In a field of pure vorticity the divergence is zero and has no deformation.

The fourth term represents deformation, the changing of the shape of a fluid element. Deformation often plays an important role in the dynamics of fronts. A field of pure deformation has no vorticity and is nondivergent. Unlike divergence and vorticity, deformation is not rotationally invariant; that is, measurements of it depend upon the orientation of the coordinate system [3].

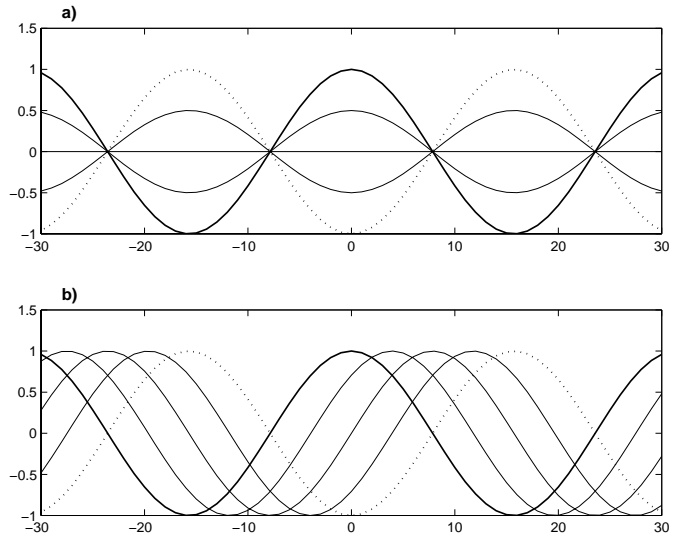
### III. RESULTS AND DISCUSSION

#### A. A traveling cosine wave

A moving feature defined by a cosine moving to the right illustrates the technique (Fig. 1). A standard weighted linear interpolation between two sampled fields shows that the feature in the interpolated field changes its amplitude, duplicates, and does not translate in space (Fig. 2a). In contrast, with the new technique, the amplitude has negligible variation, and the feature moves (Fig. 2b).



**Figure 1.** Position of the synthetic feature at time  $t = 0$  (continue line), and at time  $t = 1$  (dotted line). The abscissa axis is the distance in arbitrary units and the ordinate axis is the amplitude.

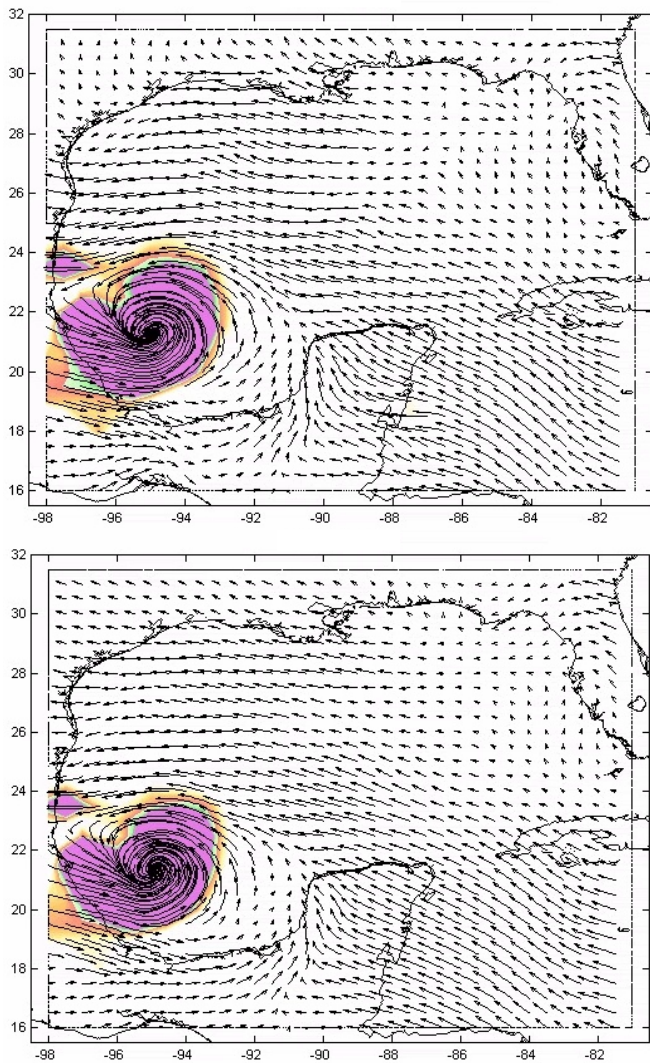


**Figure 2.** Position of the feature between two sampled times interpolated by a) weighted linear interpolation, and b) the complex empirical orthogonal function interpolation method. The thick and dotted lines represent the position of the wave at time  $t = 0$  and  $t = 1$  respectively, and the thin lines represent the estimated positions at times  $t = 1/4$ ,  $t = 1/2$ , and  $t = 3/4$ .

#### B. Generation of realistic 1-hour winds

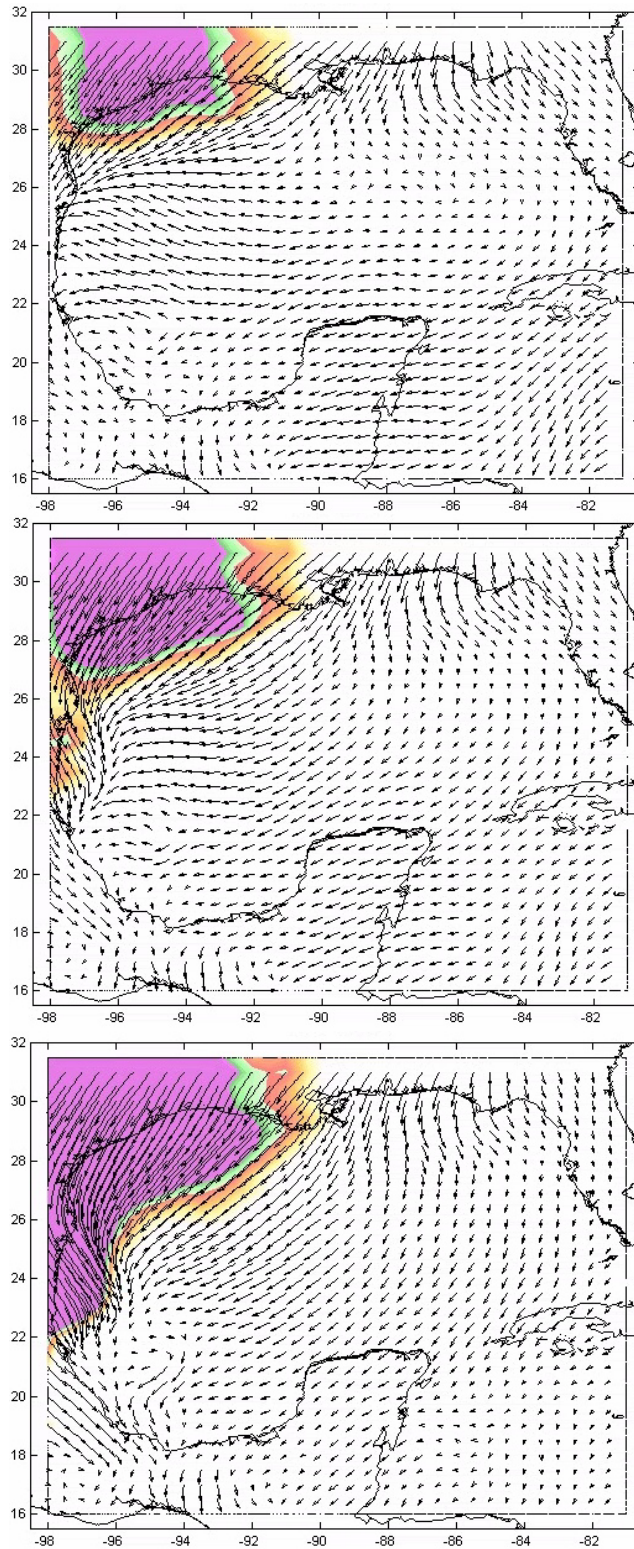
The same technique is applied to a 12-hour wind stress hybrid product that objectively uses scatterometer Qscat estimated winds from the ETA mesoscale atmospheric model

[4]. These fields will be used to force the COAPS/Florida State University Gulf of Mexico simulation [5].



**Figure 3.** Wind stress field from the hybrid (Qscat/ETA) product (upper panel) for October 4, 2000 at 12:00. In the lower panel, the wind field from the first 20 eigenmodes for the same date.

Two preliminary experiments were done, one decomposing the original winds into scalar fields, as explained in section II, and the other directly applying the new technique to the wind stress field to create the interpolated fields. It is found that using the first method the approximation in equation (1) may not be good enough when there are significant features with scales similar to that of the grid, which in this case is  $\frac{1}{4}$  of degree. The QScat often has strong gradients on those scales due to the fronts in the Gulf of Mexico so the product, although promising, is noisy and more research should be done to filter the noise from the signal.



**Figure 4.** Hourly reconstructed wind stress fields for October 8, 2000 at 0:00 (upper panel), at 6:00 (intermediate panel), and at 12:00 (lower panel).

The second experiment was performed applying the CEOF based technique to the wind stress components. The results were very successful recovering more than 90% of the variance of the original fields and features of different scales

(Fig. 3). The interpolated fields also recover moving features such as fronts (Fig. 4).

### III. CONCLUSIONS

A new technique for time and space optimal interpolation is presented. The technique is based in propagating EOFs recovering the field in eigenmodes, interpolating each mode in time and adding them. It is applied to a simple traveling cosine wave to prove that the movement of the wave is recovered and that the information of the CEOFs can be used to make a time interpolation of the position of the wave. A second experiment using realistic winds over the Gulf of Mexico shows that adding several modes may reproduce complex structures in the wind field, such as hurricanes and fronts. Also it is shown that moving features are identified and correctly interpolated in time. The technique is also useful to interpolate in space fields with moving features [6]. In this study 1-hour high-resolution wind stress fields are successfully generated.

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